UDC 535.42

A.S. Sysoyev, L.A. Nazarenko

O.M. Beketov National University of Urban Economy in Kharkiv

REFRACTION OF GAUSSIAN WAVE BEAM BY FLAT BORDER OF MONAXONIC CRISTAL

The results of the study of the wave beam refraction by the flat border of isotropic dielectric and monaxonic crystal are brought. The refraction of wave beam on the border of monaxonic crystal has a row of the specific features, related to that direction of distribution of phase (wave normal) and energy (ray) in an unusual wave does not coincide. In relation to a crystal position of his optical axis and main values of tensor of inductivity are known. For the decision of task presentation of the field of falling wave beam is used as the Fourier integral on flat waves with their subsequent analytical displacing in the parabolic approaching. Because of anisotropy there are two refracted bunches in a monaxonic crystal - usual and unusual

In a usual wave beam the wave normal of axial wave coincides with the ray vector of this wave. In an unusual beam they do not coincide in this connection, axes are perpendicular to the wave normal and not perpendicular ray. Passing is therefore done to the oblique-angled wave - ray system that descript position of power center of unusual bunch is correct.

A case is widely used in appendixes, when an optical axis lies in plane falling, here a ray lies in the same plane and a corner between a wave normal and ray is determined by a simple formula.

The change of basic parameter of beam, called the variance, is determined by the change of phase, therefore for an unusual beam it depends on a wave coordinate, but not on the ray. The operator of transformation of variance is certain on the border of division, that allows to obtain information about the structure of refracted wave beams without the decision of border task.

Knowledge of operators of transformation for the variances of refracted wave beams easily allows to define the structure of the field of the last wave beams at falling of wave beam on the arbitrary stratified structure with flat borders, containing the layers of isotropic dielectric and monaxonic crystal

Key words: wave beam, monaxonic crystal, tensor of dielectric penetration.

Introduction

Refraction wave beam at the boundary of an isotropic dielectric - uniaxial crystal has a number of specific features related to the fact that the direction of phase propagation (wave normal) and energy (the beam) are not the same in the extraordinary wave. Regarding the crystal we know the position of its optical axis and the principal values of the dielectric permittivity tensor. Changing the default setting of the beam - variance - is determined by the phase change, so it depends on the position of the wave instead of radiation for the extraordinary beam.

Main part

Let a plane boundary $z=0$ of a uniaxial crystal that fills the half-space $z > 0$, is at an angle of Gaussian wave beam with a transverse component of the electric field strength:

$$\vec{E}_1(x_1,z_1)=\vec{E}_1(n_1A_0/A_1)^{1/4}H_m\left(\sqrt{k_0n_1/A_1}\,x_1\right)\times$$
$$\times exp\left[-k_0n_1x_1^2/2V_1\right]exp\left\{i\left[k_0n_1z_1-\left(m+\frac{1}{2}\right)u_1\right]\right\} \qquad (1)$$

The wave beam (1) is distributed in a homogeneous isotropic dielectric with a refractive index n_1 that fills the half-space $z<0$ (see Fig.1).

Regarding the crystal the position of its section of the optical axis is known and the principal makes the angle ¨ with the normal $\vec{q}$ to the interface, and the principal values $£_o$ and $£_e$ of the dielectric tensor $£ = £_o + (£_e - £_o)\vec{c}\cdot\vec{c}$.

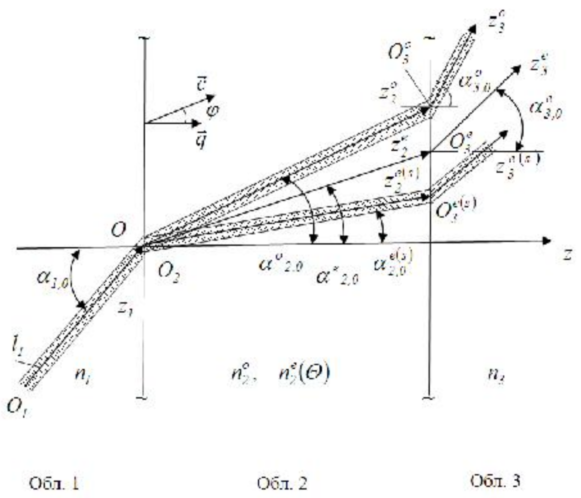

Fig.1 Distribution of wave beam in a homogeneous isotropic dielectric

The coordinate system (x_1, z_1) is related to the beam, the axis z_1 is directed along the wave beam axis, its beginning is in the beam opening. The distance of the axis of the incident beam opening along the axis to the point of its intersection with the surface of the crystal is l_1. In the formula (1) $H_m(x)$ - Hermite polynomial of the m - order, $k_0 = μ / c$, c is the speed of light in vacuum.

All geometric and partly phase parameters of the Gaussian wave beam are implicitly contained in the complex parameter

$$V_1(z_1) = n_1 A_0 + iz_1 \qquad (2)$$

called the variance wave beam $[2]$, where

$$A_0 = k_0 ´_0^2 / 2;$$

$$A_1 = k_0 n_1 ´_1^2 / 2 = (Re V_1)(1 + D_1^2);$$

$$R_1 = (JmV_1)(1 + D_1^{-2})$$

$$D_1 = tg\, u_1 = (JmV_1) / ReV_1,$$

where $´_0$ is the radius of the beam spot of the field in the opening, $´_1$ is the radius of the spot of the field, R_1 is the radius of curvature of the wave beam surface, u_1 is an additional phase shift of the beam due to its spreading during propagation.

The field (1) is a quasi-optical approximation of the field

$$\vec{E}_1(x_1, z_1) = \vec{E}_1 \frac{k_0 n_1}{2®} \int_{-\infty}^{\infty} F_m(¶_1) \times \exp\left[ik_0 n_1\left(¶_1 x_1 + \sqrt{1 - ¶_1^2}\, z_1\right)\right] d¶_1, \qquad (3)$$

where

$$F_m(¶_1) = (2®A_0 / k_0)^{1/2} (-i)^m \exp\left(-k_0 n_1^2 A_0 ¶_1^2 / 2\right) \times H_m\left(n_1 \sqrt{k_0 A_0}\, ¶_1\right),$$

$F_m(¶_1)$ - Fourier - transformation of the function of the field distribution of the incident beam in the opening when $z_1 = 0$, $¶_1 = sin \;\; _1$.

The formula (3) represents the expansion of the field of the incident wave beam (1) in a Fourier integral in plane waves, $\;_1$ are the corners that make up the wave vectors $\vec{k}_1 = k_0 n_1 \vec{n}_1$ of the waves with the beam axis z_1, $\vec{n}_1$ are single wave normals of plane waves.

The angles of incidence of plane waves on the surface of the crystal are $\sim_1 = \sim_{1,0} + \;_1$. Here and further the first lower indices of the parameters of wave beams indicate spatial region, and the second lower index 0 indicates that the value refers to a plane wave propagating along the axis of the wave beam (axial wave).

For the quasi-optical approximation in the formula (3) $\sqrt{1 - ¶_1^2}$ must be expanded up to quadratic terms and minimize the resulting integral analytically. As a result, we get the formula (1).

Due to the anisotropy in the uniaxial crystal there are two refracted beams - ordinary $\vec{E}_2^o$ and extraordinary $\vec{E}_2^e$, defined by the formula:

$$\vec{E}_2^j(x_2^j, z_2^j) = \frac{k_0 n_1}{2®} \int_{-\infty}^{\infty} \vec{E}_2^j(¶_1) L_1(¶_1) \times \exp\left[ik_0 n_2^j\left(¶_2^j x_2^j + \sqrt{1 - (¶_2^j)^2}\, z_2^j\right)\right] d¶_1, \qquad (4)$$

where

$$j = o, e;\; L_1(¶_1) = \exp ik_0 n_1 \sqrt{1 - ¶_1^2}\, l_1;$$

$$n_2^o = \sqrt{£_o}$$

$$n_2^e = n_2^e(n) = \left[£_o £_e / \left(£_o \sin^2 n + £_e \cos^2 n\right)\right]^{1/2};$$

$$¶_2^j = sin \;\; _2^j;$$

$\;_2^j$ are corners that make up the wave vectors $\vec{k}_2^j = k_0 n_2^j \vec{n}_2^j$ of the plane waves in the expansion (4) field of refracted beams in plane waves with axes z_2^j coinciding with the axes of refracted beams; n is the angle between the wave vector $\vec{k}_2^e$ and the optical axis of the crystal, it is a function of the variable of integration $¶_1$. The factor $L_1(¶_1)$ describes the spreading of the beam path l_1.

The angles of refraction $\sim_2^j$ of plane waves in the expansion (4) defined by the formula $\sim_2^j = \sim_{2,0}^j + \ _2^j$, where $\sim_{2,0}^j$ are the angles of refraction of plane waves with wave vectors $\vec{k}_{2,0}^j$ coinciding with the axes of wave beams.

Using the laws of refraction

$$n_1 \sin\sim_1 = n_2^j \sin\sim_2^j,$$

$$n_1 \sin\sim_{1,0} = n_2^j \sin\sim_{2,0}^j \qquad (5)$$

it is easy to get

$$¶_2^j = \left(n_1 / n_2^j\right) t_{12}^j ¶_1 + o\left(¶_1^2\right), \qquad (6)$$

where

$$t_{12}^j = \cos\sim_{1,0} / \cos\sim_{2,0}^j \qquad (7)$$

$n_2^e = n_2^e(\text{n}_o)$; n_o is the angle between the optical axis and the wave vector $\vec{k}_{2,0}^e$ of the extraordinary beam coinciding with its axis. Changing places at the lower indices of $t_{\sim}^j$ in the future will mean replacing the numerator and denominator in the formula (7).

The amplitudes of the reflected and refracted wave beams are identical to those for plane waves, their definition is given in $[3]$.

In an ordinary wave beam the wave normal $\vec{n}_{2,0}^o$ of axial wave coincides with the axial wave vector $\vec{s}_{2,0}^o$ of this wave beam. In an extraordinary beam the wave normal $\vec{n}_{2,0}^e$ of the axial wave and the radial vector $\vec{s}_{2,0}^e$ does not coincide in direction. In general, when the optical axis does not lie in the plane of incidence, the ray vector $\vec{s}_{2,0}^e$ along which the energy transfers in the beam does not lie in the plane of incidence either. In applications, the case when the optical axis lies in the plane of incidence (the main plane of incidence) is commonly used. In the main plane of incidence, the refractive index for extraordinary waves varies with the direction of the maximum. At the same time the beam lies in the same plane and the angle ¢ between the wave normal $\vec{n}_{2,0}^e$ and the line $\vec{s}_{2,0}^e$ is defined by the formula

$$tg¢ = \pm \frac{|£_e - £_o| tg«}{£_e + £_o tg^2«}, \qquad (8)$$

where « is the angle between the vectors $\vec{n}^e$ and $\vec{c}$.

In the formula (8) + sign is taken if the vector is between the normal to the surface.

In the formula (4) two coordinate systems $\left(x_2^j, z_2^j\right)$ are introduced whith the axes z_2^j directed along the wave vectors $\vec{k}_{2,0}^j$ (along the axis of wave beams), and the origin of coordinates 0_2^j is at the interface. In the future, the coordinate system whith the axis z directed along the wave vector, we will call wave, and the system with the axis z along the beam we will call $\vec{s}_{2,0}^e$ - beam.

The direction of the wave beam axis must coincide with the direction of the energy flow. The coordinate value $x_2^j = 0$ in the formula (4) defines the position of the energy center of the beam. For the ordinary wave beam the axis $x_2^o \perp \vec{s}_{2,0}^o$ and the value $x_2^o = 0$ correctly describes the position of the energy center of an ordinary beam at any coordinate z_2^o (t. E. energy transfers along z_2^o). For the extraordinary wave beam the axis x_2^e is not perpendicular to $\vec{s}_{2,0}^e$ and therefore the co-ordinate axis x_2^e needs to move to the axis $x_2^{e(s)} \perp \vec{s}_{2,0}^e$, i.e. the wave from the rectangular coordinate system $\left(x_2^e, z_2^e\right)$ should move in oblique wave-beam $\left(x_2^{e(s)}, z_2^e\right)$, which is

$$x_2^{e(s)} = x_2^e \cos¢ - z_2^e \sin¢ \qquad (9)$$

For beams with poor divergence the function $F_m(¶_1)$ has sharp local maximum in the vicinity $¶_1 = 0$, and therefore, by substituting (9) into (4), $¶_2^e tg¢$ can be neglected in comparison with $\sqrt{1 - \left(¶_2^e\right)^2}$ (for all of crystals $tg¢ << 1$). As a result, for the extraordinary beam we receive

$$\vec{E}_2^e\left(x_2^e, z_2^e\right) = \frac{k_o n_1}{2®} \int_{-\infty}^{\infty} \vec{E}_2^e(¶_1) F_m(¶_1) L_1(¶_1) \times$$
$$\times \exp\left\{ i k_0 n_2^e(\text{n}) \left[¶_2^e x_2^{e(s)} / \cos¢ + \sqrt{1 - \left(¶_2^e\right)^2} z_2^e \right] \right\} d¶_1 \qquad (10)$$

Displacing the integrals (4) for the ordinary beam and (10) for the extraordinary in the quasi-optical approximation we finally get

$$\vec{E}_2^j\left(\bar{x}_2^j,\bar{z}_2^j\right)=\vec{E}_2^j\left(n_2^j A_0 / A_2^j\right)^{1/4} H_m\left(\sqrt{k_0 n_2^j / A_2^j}\,\bar{x}_2^j\right)\times$$
$$\times exp\left[-k_0 n_2^j\left(\bar{x}_2^j\right)^2 / 2V_2^j\right] exp\left\{i\left[k_0 n_2^j z_2^j-\left(m+\frac{1}{2}\right)u_2^j\right]\right\} \qquad (11)$$

where

$$\bar{x}_2^o = x_2^o, \bar{x}_2^e = x_2^e - z_2^e tg¢\ ,\ \left(A_2^j\right)^{-1} = Re\left(V_2^j\right)^{-1},$$
$$V_2^j\left(z_2^j\right)=V_2^j(0)+iz_2^j$$
$$V_2^j(0)= n_2^j A_o\left(t_{21}^j\right)^2 + i\left(n_2^j / n_1\right)\left(t_{21}^j\right)^2 l_1 . \qquad (12)$$

Thus, in the extraordinary wave beam the phase of the wave propagates along the normal direction (along the axis z_2^e), and energy – along the straight line $x_2^e = z_2^e tg¢$, i.e. along the beam. Changes in the variance beam (12) are determined by the phase change, so it depends on the wave coordinates but not from ray coordinates.

Formula (11) for the extraordinary wave beam coincides with the similar formula for work, which it obtained from the solution of parabolic differential equations. However, this formula is obtained for the space which is completely filled with a crystal but in this work it is obtained for the boundary value problem, for half-space filled with a crystal in the breaking of them falling on the boundary of the wave beam. This allows obtaining a conversion algorithm variance beam *V* when crossing the boundary. Comparing formulas (2) for $V_1(l_1)$ and (12) for $V_2^j(0)$ we can record

$$V_2^j(0)= B_1^j V_1(l_1), \qquad (13)$$

where B_1^j is the conversion operators of variances of the beams when crossing the border line from the 1st circumference in the 2nd circumference determined by the formula

$$B_1^j = \left(n_2^j / n_1\right)\left(t_{21}^j\right)^2 \qquad (14)$$

If the crystal occupies not all the half-space $z > 0$, but only part of it and is restricted by a flat surface (2nd border line), which has isotropic circumference outside with the refractive index n_3, the last wave beams in area 3 is defined by the formula

$$\vec{E}_3^j\left(x_3^j,z_3^j\right)=\frac{k_0 n_1}{2®}\int_{-\infty}^{\infty}\vec{E}_2^j\left(¶_2^j\right)F_m(¶_1)L_1(¶_1)L_2^j\left(¶_2^j\right)\times$$
$$\times exp\left[ik_0 n_3\left(¶_3^j x_3^j+\sqrt{1-\left(¶_3^j\right)^2}\,z_3^j\right)\right]d¶_1 \qquad (15)$$

where the factor $L_2^j\left(¶_2^j\right)= exp\left[ik_0 n_2^j\sqrt{1-\left(¶_2^j\right)^2}\,l_2^j\right]$ describes the blurring (diffraction) beams on the path l_2^j;

l_2^j - the distance along the wave normals $\vec{n}_{2,0}^j$ (along the axes of the beams z_2^j from the first border line to the second), $¶_3^j = sin\ _3^j$; $_3^j$ - the angles that make up the wave vectors $\vec{k}_3^j = k_o n_3 \vec{n}_3^j$ of plane waves in (15) with the axes z_3^j . The direction of the axis z_3^j of the wave coordinate system coincides with the direction of the wave vectors $\vec{k}_{3,0}^j$ and is determined from the laws of refraction on the second border line

$$n_2^j\ sin\simeq_2^j = n_3\ sin\sim_3^j\ ;\ n_2^j\ sin\simeq_{2,0}^j = n_3\ sin\sim_{3,0}^j \qquad (16)$$

from which $¶_3^j = \left(n_2^j / n_3\right)t_{23}^j ¶_2 = \left(n_1 / n_3\right)t_{12}^j t_{23}^j ¶_1 + o\left(¶_1^2\right)$,

where $t_{23}^j = cos\simeq_{2,0}^j / cos\sim_{3,0}^j$; $\simeq_{2,0}^j$ - the angle of incidence of the axial wave at the second border line. If the boundaries are parallel $t_{12}^j t_{23}^j = t_{13}^j = cos\sim_{1,0} / cos\sim_{3,0}^j$.

In an isotropic medium over a layer of the crystal $\vec{n}_{3,0}^o = \vec{s}_{3,0}^o$, $\vec{n}_{3,0}^e\ II\ \vec{s}_{3,0}^e$, and therefore the axis z_3^e is parallel to the axis $z_3^{e(s)}$.

Laws of refraction (16) provide that the extraordinary axis of the last extraordinary wave beam in an area 3 matches z_3^e but they do not take into account the direction of the flow of energy, and the energy center of this beam extends along the axis $z_3^{e(s)} II\ z_3^e$ (see Fig.1). Therefore, due to the parallelism of these axes, in the formula (15) for the extraordinary wave beam ($j = e$) it is necessary to do the replacement $z_3^e \to z_3^{e(s)}$ и $x_3^e \to x_3^{e(s)}$, i.e. go from a wave into the ray coordinate system. Phases in this transition have no effect, but it gives the correct position of the energy center of the beam.

The result is

$$\vec{E}_3^j\left(\bar{x}_3^j,\bar{z}_3^j\right)=\vec{E}_3^j\left(n_3 A_0 / A_3^j\right)^{1/4} H_m\left(\sqrt{k_0 n_3^j / A_3^j}\,\bar{x}_3^j\right)\times$$
$$\times exp\left[-k_0 n_3\left(\bar{x}_3^j\right)^2 / 2V_3^j\right] exp\left\{i\left[k_0 n_3 \bar{z}_3^j-\left(m+\frac{1}{2}\right)u_3^j\right]\right\} ,(17)$$

where

$$\bar{x}_3^o = x_3^o,\ \bar{z}_3^o = z_3^o;\ \bar{x}_3^e = x_3^{e(s)},\ \bar{z}_3^e = z_3^{e(s)};$$
$$\left(A_3^j\right)^{-1} = Re\left(V_3^j\right)^{-1},$$

$$V_3^j(\bar{z}_3^j) = V_3^j(0) + i\bar{z}_3^j;$$
$$V_3^j(0) = n_3^j A_o (t_{31}^j t_{21}^j)^2 + \qquad (18)$$
$$+ i[\,(n_3/n_1)(t_{32}^j t_{21}^j)^2 l_1 + (n_3/n_2^j)(t_{32}^j)^2 l_2^j\,]$$

If the boundaries are parallel to $t_{32}^j t_{21}^j = t_{31}^j$. Comparing formula (18) for $V_3^j(0)$ with formula (12) for $V_2^j(l_2^j)$ we can record

$$V_3^j(0) = B_2^j V_2^j(l_2^j), \qquad (19)$$

where the conversion operator of the variance of the wave beam on the second interface, is defined by the formula

$$B_2^j = (n_3/n_2^j)(t_{32}^j)^2 \qquad (20)$$

This passage makes it easy to determine the field structure of the wave beam while passing them an arbitrary layered structure with flat interfaces composed of isotropic dielectric layers and layers of uniaxial crystals. On the k - interface the conversion of the variance of the beam is carried out according to the rule

$$V_{k+1}^j(0) = B_k^j V_k^j(l_k^j), \quad B_k^j = (n_{k+1}^j/n_k^j)(t_{k+1,k}^j)^2,$$
$$V_{k+1}^j(l_{k+1}^j) = V_{k+1}^j(0) + il_{k+1}^j.$$

where l_k^j is the distance between the *k-1* and *k*-boundaries measured along of the wave coordinate z_k^j

Knowledge of the structure of the field of wave beams that have passed the layer of anisotropic dielectric is necessary for the description of fields in open resonators with anisotropic dielectrics.

Conclusion

Conversion operators for variance beam while passing through the anisotropic layer are defined which makes it easy to determine the structure of the field of the refracted wave beams when the beam wave passes through a random layer structure with flat boundaries.

References

1. Vaganov R.B / R.B. Vaganov // Technology and Electronics. - 1969 - vol.14, №3. - p. 393-402.

2. Deschamps J. /J. Deschamps, P. Mast // Coll. "Quasioptics". - Moscaw. World, 1966 - p. 198-209.

3. Fedorov F.I / F.I Fedorov, VV Filippov // Reflection and refraction of light transparent crystals. - Minsk; Science and Technology, 1976.

4. Goncharenko AM /A.M. Goncharenko //Gaussian beams of light. - Minsk; Science and Technology 1977.

Литература

1. Ваганов Р.Б. /Р.Б. Ваганов// Радиотехника и электроника. – 1969. – т.14, вып.3. – С. 393-402.

2. Дешан Ж., Маст П. Сб. "Квазиоптика". – М. Мир, 1966 – с. 198-209.

3. Федоров Ф. И./Ф.И.Федоров, В.В. Филиппов //. Отражение и преломление света прозрачными кристаллами. – Минск; Наука и техника, 1976.

4. Гончаренко А.М. /А.М. Гончаренко // Гауссовы пучки света. – Минск; Наука и техника, 1977.

Рецензент: доктор техн. наук проф. Говоров Ф.П., Харьковский национальный университет городского хозяйства имени А.Н. Бекетова

Автор: СЫСОЕВ Анатолий Сергеевич

Канд. физ.- мат. наук, доцент, доцент кафедры физики Харьковского национального университета городского хозяйства имени А.Н. Бекетова.

E-mail: anatolii_sysoev@mail.ru

Автор: Назаренко Леонид Андреевич

Доктор техн. наук, профессор, зав.кафедри СИС Харьковского национального университета городского хозяйства имени А.Н. Бекетова .

ЗАЛОМЛЕННЯ ГАУСОВОГО ХВИЛЬОВОГО ПУЧКА ПЛАСКОЮ МЕЖЕЮ ОДНООСНОГО КРИСТАЛУ

А.С. Сисоєв, Л.А. Назаренко

Наведено результати дослідження заломлення хвильового пучка пласкою межею ізотропного діелектрика та одноосного кристалу. Визначений оператор перетворення основного параметру хвильового пучка, який має назву варианса, при проходженні межі розподілу ізотропний діелектрик – одноосний кристал. Це дозволяє, не розв'язуючи граничної задачі, отримати інформацію про структуру заломлених хвильових пучків.

Ключові слова: *хвильовий пучок, одноосний кристал, тензор діелектричної проникності.*

ПРЕЛОМЛЕНИЕ ГАУССОВОГО ВОЛНОВОГО ПУЧКА ПЛОСКОЙ ГРАНИЦЕЙ ОДНООСНОГО КРИСТАЛА

А.С. Сисоєв, Л.А. Назаренко

Приведены результаты исследования преломления волнового пучка плоской границей изотропного диэлектрика и слоем одноосного кристалла. Определен основной параметр волновых пучков, называемый вариансом, при прохождении слоя одноосного кристалла, что позволяет не решая граничной задачи получить информацию о структуре преломленных волновых пучков.

Ключевые слова: *волновой пучок, одноосный кристалл, тензор диэлектрической проницаемости.*