Mathematical model of consumer regulators management for alignment of electric load graphs of transformer substation 10/0.4 kV

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Abstract -The change of load of the transformer substation TS 10 / 0,4 kV in the mode of maximum and minimum load is measured. It is noted that the process is non-stationary at the daily interval. Hourly intervals of time are highlighted on the daily graph of electrical load, the study of the law of distribution is conducted, and the normal law of distribution is confirmed by Pearson's criterion. The stationarity test was performed on parametric tests, namely the verification of the Fisher's constant variance and the mathematical expectation of the Student's t-test and the correlation function. *The values of numerical characteristics were obtained at the stationary areas and a probabilistic mathematical model of the TS 10 / 0.4 kV load was constructed.*

The method of equalization of the electric load graph of TS 10 / 0,4 kV is developed by optimal management of the power of consumers-regulators in urban electric networks, taking into account the features of the electric load graph of the main consumers of TS 10 / 0,4 kV. For this purpose, we set a target function, where the criterion of optimization is the total minimum cost to cover the losses of electricity in the network caused by the irregularity of the electric load graph and the electricity consumed, a system of constraints caused by the load capacity of the transformers and the required energy consumption by the technology regulators has been compiled and boundary conditions determined by the installed capacities of the consumers-regulators. In order to take into account the trade-off between the constituents of the objective function, the weighted coefficients of the generalized objective function are chosen by the method of expert estimation. A nonlinear programming method was used to fulfill the optimization process, and the extremum of the function was found using the Newton method. The solution of the problem is implemented in MS Excel.

Keywords- *electrical load graph, distribution networks, probabilistic mathematical model, nonlinear programming, optimization, consumer controllers*

I. FORMULATION OF THE PROBLEM

One of the ways of increasing the efficiency of the electrical distribution networks (EDN) is the alignment electrical load graph (ELG). An analysis of the methods and means of regulating the ELG indicates that the coverage of the ELG occurs mainly at the higher levels of the electricity system (ES), while the most effective influence is precisely on the process of electricity consumption in the distribution electrical networks. Formation of uniform modes of power consumption will allow to increase efficiency of production, transmission and consumption of electricity. Improv-

ing the configuration of the ELG at lower ES levels will also influence the alignment of electrical load graph at higher ES levels to some extent. The scientific substantiation of methods, criteria and technical means of control of the modes of consumption of consumers-regulators (CRs) in the distribution electric networks for the purpose of alignment of the electrical load graph is an urgent scientific task.

II. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Analysis of recent research and publications indicates the problem of non-uniformity of electrical load graph in distribution electrical networks.

Many scientists, including Zorin V.V., Kirilenko O.V., Yandulsky O.S., Govorov P.P., Mikhailov V.V., Rosen V.P. et al. addressed the issue of increasing the efficiency of electrical networks. The papers considered the optimization of circuits, parameters and modes in distribution networks. The issue of involving a group of consumers-regulators was investigated in order to change the configuration of electric load graphs. The issue of automatic selection of the optimal structure and modes of operation of consumers-regulators, which ensure the formation of an electric load graphs, has not been resolved.

III. THE PURPOSE OF THE ARTICLE

The purpose of the article is to increase the efficiency of distribution of power networks by aligning the TS 10/0.4 kV electrical load graph with the management of consumers-regulators.

IV. BASIC MATERIAL

In order to improve the automatic control systems of power capacity of additionally connected to TS 10 / 0,4 kV CR, it is important to forecast the hourly electrical load of the main consumers of TS 10 / 0,4 kV within one day, i.e. to make an operational forecast [1].

A prerequisite for an objective mathematical prediction is the availability of an adequate mathematical model of the 10 / 0.4 kV TS mode, for which different methods are used [2]. Given the task, it is most appropriate to apply the probabilistic method.

Given that the argument of a random function is time t , the electrical load graph (ELG) is the implementation of a random process of power change.

The process of power consumption of the main consumers of 10 / 0.4 kV TS can be represented by a set of randomly ordered random variables $P(\tau_1), P(\tau_2), \dots, P(\tau_n)$, where $\tau_i < \tau_{i+1}$. In what follows, we assume that the moments of measurement τ_i equidistant from each other by magnitude $\Delta\tau$, that is, for any i , is $\tau_{i+1} = \tau_i + \Delta\tau$.

The researches of random process $P_i(\tau_i)$ of the change of load at TS 10 / 0.4 kV were conducted, which is shown in fig. 1.

Fig. 1. Implementation of a random function load TS 10 / 0.4 kV

The change of load of the transformer substation TS 10 / 0.4 kV is stochastic, which is non-stationary at the daily interval. In a non-stationary random process, it is possible to determine the characteristic periods of time of the stationary areas. The stationary random process can be analyzed from any point in time because it is in a state of probable equilibrium, that is, the numerical characteristics of random variables $P(\tau_i)$ are independent of time τ_i . The separation and study of the stationary process were performed at hourly intervals. An estimate of the distribution law is made for each interval. It is found that the distribution law is normal at all intervals (Pearson-tested (χ^2)).

The conditions of stationarity of a random process are [3]:
the constancy of the process variance

$$D_p(t) = D_p = \text{const}; \quad (1)$$

- 1) the constancy of the mathematical expectation of the process

$$M_p(t) = M_p = \text{const}; \quad (2);$$

- 2) dependence of the correlation function on one argument, what is the difference between the arguments

$$K_p(\tau_i, \tau_i + \Delta\tau) = K_p(\tau). \quad (3)$$

The time series are divided into two levels by the number of values of the part, each of which is considered as some independent sample set having a normal distribution law. Let the first part $P^{(I)}$ contains n_I observations $P(\tau_i)$, $i = 1, 2, \dots, n_I$, and the other part – $P^{(II)}$ contains n_{II} observations $P(\tau_i)$, $i = n_I + 1, \dots, n_I + n_{II}$. For each part of the time series, calculate the sample mean M_{pI}, M_{pII}

$$M_{pI} = \frac{1}{n} \sum_{i=1}^n p_i \quad (4)$$

and sample variances D_I, D_{II} .

$$D_I = \frac{1}{n_I - 1} \cdot \sum_{i=1}^{n_I} (p_i - M_{pI})^2. \quad (5)$$

To confirm that the variances of both parts of the series are the same, we use the Fisher test:

$$F_S = \frac{\max(D_I, D_{II})}{\min(D_I, D_{II})}, \quad (6)$$

where D_I, D_{II} – variance estimates calculated from the first one n_I and the other n_{II} parts of the time series.

The critical area is the interval (F_{kp}, ∞) . If we observe a value F_S does not fall into the critical region, then the null hypothesis is accepted $H_0: D_I = D_{II}$. As an example of a value on an interval 03:00-04:00 make up $F_S = 1,138 < F_{kp} = 1,476$.

As a result of the calculations, it is determined that the actual values of the Fisher coefficients do not exceed the critical values at the selected intervals, that is, the hypothesis of the constancy of the variances is accepted.

Then the hypothesis about the constancy of the mean of the time series is tested. We calculate the value of the Student's test

$$K_S = \frac{M_{pI} - M_{pII}}{\sqrt{(n_I - 1)D_I + (n_{II} - 1)D_{II}}} \cdot \sqrt{\frac{n_I \cdot n_{II}(n_I + n_{II} - 2)}{n_I + n_{II}}}. \quad (7)$$

If inequality holds $K_S < t(1 - \alpha, n_I + n_{II} - 2)$, then the hypothesis of the constancy of mathematical expectation is accepted with a level of significance α .

We calculate the value of the Student's test and the critical two-sided. For example, the critical area for maximum load mode is shown in the time interval 03:00-04:00, is a union of two intervals and looks like $(-\infty, -1,976] \cup [1,976, \infty)$. The value of t-statistics $K_S = -0,233$ does not fall into this region, which testifies to the homogeneity (heteroscedasticity) of residuals of the random distribution and the hypothesis is accepted $H_0: M(P(\tau_i)) = \text{const}$ about equality of mathematical expectations.

Acceptance of hypotheses about the equality of variances and equality of mathematical expectations allows us to accept the hypothesis about the absence of a trend component in a given time series, that is, a hypothesis about the stationarity (in the broad sense) of the time series.

The degree of statistical association between the sequences $P(\tau_1), P(\tau_2), \dots, P(\tau_n)$ and $P(\tau_{1+l}), P(\tau_{2+l}), \dots, P(\tau_{n+l})$, that are displaced relative to each other by lag l can be determined by the autocorrelation coefficient:

$$\rho(l) = \frac{(P(\tau_i) - M[P(\tau_i)])(P(\tau_{i+l}) - M[P(\tau_{i+l})])}{D} \quad (8)$$

The sequence of autocorrelation coefficients $\rho(0), \rho(1), \rho(2)$ is an autocorrelation function of the time series, graphs of dependencies of values $\rho(l)$ from the magnitude of the lag for the interval 03:00-04:00 built in STATISTICA shown in fig. 2.

Correlation functions indicate the ergonomics of the process and are approximated by analytical dependence (Fig. 3)

$$R(\tau) = D \cdot e^{-\alpha\tau} \cdot \cos(\beta\tau), \quad (9)$$

where D is the variance of a random function;
 α і β – least squares coefficients [4].

Fig. 2. Autocorrelation functions at the daily interval from 03:00 to 04:00 hours: a) sequence of autocorrelation coefficients; b) an approximated function

The analysis of autocorrelation functions makes it possible to conclude that time series are stationary at all intervals, for both maximum and minimum load modes.

For the stationary process, comprehensive statistical information on load modes is the two simplest numerical characteristics - mathematical expectation and standard deviation [5 – 9].

For a reliable description of the load change processes TS 10/0,4 kV probabilistic modeling method was used. Using the measurement data, the values of the numerical characteristics of the studied processes were obtained at the stationary sites [10] and a probabilistic mathematical model of loading was constructed. The calculated value of the load is found by the formula:

$$P_i(t) = M[P_i(t)] + \beta\sigma[P_i(t)], \quad (10)$$

where $M[P_i(t)]$ - mathematical expectation of load at time t ,

$\sigma[P_i(t)]$ – standard deviation of load at time t ,

β – coefficient of scatter of random load magnitude, which takes into account the intensity of scatter of random load values near the accepted design value.

Probabilistic mathematical model of loading TS 10/0,4 kV in the mode of maximum (Fig. 3) and minimum (Fig. 4) loads.

Fig. 3. Probabilistic mathematical model of loading TS 10/0,4 kV in the maximum load mode: a) active, b) reactive load

Fig. 4. Probabilistic mathematical model of loading TS 10/0,4 kV in the mode of minimum loads: a) active, b) reactive load

The simulated ELGs allow to take into account the real process of load behavior, which is an important task for performing qualitative modeling of the work of consumers-regulators.

For the management of the ELG, it is necessary to have a legal mechanism of regulation, namely, the existence of the Law of Ukraine "On the Electricity Market" [15] and economic incentives for the regulation of the ELG, which provide a change in the price of electricity (EP) by hours of day and seasonality.

The level of technical loss in electrical networks depends on the losses in lines and step down transformers 10 / 0,4 kV, i.e. on the nature and magnitude of the change of the ELG.

Thus, the solution to the problem of increasing the efficiency of the power supply system [13], i.e. optimization of the mode is reduced to control the process of power consumption of CR.

Considering the mode of operation of TS 10 / 0,4 kV, we will define that under the main load of TS 10 / 0,4 kV we will understand the unregulated load of consumers connected to TS 10 / 0,4 kV, and under the concept of load of consumers-regulators (CR) - attached load of a special group CR that force can be controlled

The CR management process can be considered as a mathematical problem. The mathematical model of the optimization problem includes the objective function, constraints and boundary conditions.

Load optimization criterion TS 10/0,4 kV is the total minimum cost of the additional costs of the energy supply organization to cover the losses of electricity caused by the unevenness of the ELG and for the consumed electricity to carry out the technological process of CR.

The optimized parameters are the power of the CR at the i -th intervals (P_{CRi}). The coefficients for the CR load optimization problem are tariff coefficients (k_i), that are valid for a given period of time.

It is necessary to minimize the objective function that takes into account the technical component - the loss of electricity and the economic component - the cost for the consumption of Electric Power (EP), taking into account the tariff coefficients

$$B = T \cdot \left(\sum_{n=1}^N \sum_{i=1}^T \left(\frac{(P_i(t) + P_{CPI})^2 + Q_i^2}{U_n^2} \cdot r_n \cdot t_i \right) + \sum_{j=1}^J \sum_{i=1}^T \left(\frac{\Delta P_{K3} \cdot ((P_i(t) + P_{CPI})^2 + Q_i^2)}{m \cdot S_{HOM}} \right) \right)$$

$$+m \cdot \Delta P_{xx} \cdot t_i) + T \cdot \sum_{i=1}^N (k_i P_{CPi} t_i) \rightarrow \min \quad (11)$$

under the constraints given in the form of inequality and equation

$$\begin{cases} P_{CPi} \leq (m \cdot S_{\text{HOMTII}} - \sqrt{P_i(t)^2 + Q_i(t)^2}) \cdot \cos \varphi \\ \sum_{i=1}^N P_{CPi} \cdot t_i = W_{CP} \end{cases} \quad (12)$$

and boundary conditions that define the range of change in the values of the variables:

$$0 \leq P_{CPi} \leq P_{\text{HOMCP}}, \quad (13)$$

where $\cos \varphi$ – coefficient of phase shift;

$\sqrt{P_i(t)^2 + Q_i(t)^2}$ – load of main consumers of TS in the i -th period of time (determined from the probabilistic model);

P_{HOMCP} – rated power of CR;

W_{CP} – electricity for CR needs.

Considering the disproportionate components of the objective function (a much smaller proportion of the cost of covering electric power compared to the cost of consumed electric power for the needs of the CR), it is important to take into account the scientifically sound proportions between these values, which is an important factor in the effective regulation of the ELG. Increasing the share of losses component allows to change the configuration of the ELG most effectively.

It is possible to make a compromise between taking into account the data of the constituents of the objective function, taking into account the weight coefficients. Multicriteria optimization is carried out according to a generalized objective function, which includes the accepted criteria with their weighting factors. The generalized function is written as follows:

$$B_{\text{gen}} = \sum_{k=1}^K \frac{\alpha_k B_k}{B_{k\text{norm}}} \rightarrow \min, \quad (14)$$

where B_k – k -th the target function expressing k -th criterion;

$B_{k\text{norm}}$ – the normalized value of the k -th objective function;

α_k – the weight factor of the k -th objective function;

K – number of criteria accepted.

Dividing the k -th objective function by its normalized value brings all the objective functions to one dimension (relative units).

Consider several variants of the coefficients obtained by the method of expert estimates. From the values of the weighting coefficients $\alpha_1 = 0,83$, $\alpha_2 = 0,17$, the optimization function becomes sensitive to the technical component – the level of loss of electrical energy. From the values of the weighting coefficients $\alpha_1 = 0,86$, $\alpha_2 = 0,14$, the optimization function begins to gradually decrease the sensitivity to the increased tariff coefficients of the objective function. Therefore, according to the method of expert estimations, the values of the weighting coefficients are given

$\alpha_1 = 0,85$, $\alpha_2 = 0,15$, as those for which the optimization function is the most sensitive to the two components.

Methods for solving optimization problems refer to operations research methods or mathematical programming methods [16, 17], which allow to determine the extreme value of the objective function (11) under the established constraints (12) in the range of variables determined by the boundary conditions (13).

Given that the optimization problem, that is, the problem of choosing the optimal CR power at the i time interval, is set as a nonlinear optimization problem, then a method of nonlinear programming can be applied to obtain its solution.

To determine the extremum of a nonlinear function, we use gradient methods, namely, we apply the Newton method. The solution of the optimization problem is implemented in the MS Excel package.

According to the optimization model (11 – 13), the value of the power of the CR was determined and the general ELG of TS 10/0,4 kV after the connection of the CR was constructed (Fig. 5).

Fig. 5. Electric load graph: a) consumers-regulators; b) TS 10/0,4 kV after connection of consumer regulators

Comparison of changes of electric load graphs before and after connection of consumers-regulators is made. Thus, the coefficient of non-uniformity before the connection CR $k_{n1} = 0,23$, and after connection of CR $k_{n2} = 0,42$; form factor $k_{f1} = 1,084$, $k_{f2} = 1,014$; maximum coefficient $k_{m1} = 1,79$, $k_{m2} = 1,36$; demand factor $k_{d1} = 0,46$, $k_{d2} = 0,57$; coefficient of fulfillment factor $k_{ff1} = 0,56$, $k_{ff2} = 0,74$ and load factor TS $k_{l1} = 0,26$, $k_{l2} = 0,42$, which indicates the alignment of the 10 / 0.4 kV TS ELG after connection and control of the CR load [18]. By improving the ELG configuration and increasing the efficiency taking into consideration the uniform and optimal load throughout the day, a reduction in power losses of 9.5% per unit of transmitted power has been achieved

V. CONCLUSIONS

The probabilistic mathematical model of the 10 / 0.4 kV TS load is improved, which takes into account the stationary areas and allows to increase the completeness of information support of the control process.

The method of alignment of the graph of electric load of TS 10/0,4 kV by optimal control of the power of CR is developed. To this end, a mathematical model has been created that meets the objectives of optimizing CR power management based on the selection and acceptance of the cost criterion. The technical criterion is taken into account as the cost of covering electricity losses in the network caused by the non-uniformity of the ELG, and the econom-

ic criterion as the cost of consumed electricity for the needs of CR, a system of constraints caused by the load capacity of transformers and the required amount of electricity to perform the technological process, figured out the capacity of the CR. The trade-offs between the constituents of the objective function are taken into account by the weighting coefficients of the peer review method.

The nonlinear programming method was used to fulfil the optimization process, and the relative extremum of the function was found using the Newton method. The solution of the problem is implemented in MS Excel.

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Математична модель керування споживачами-регуляторами для вирівнювання графіків навантаження ТП 10/0,4 кВ

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Проведено вимірювання зміни навантаження трансформаторної підстанції ТП 10/0,4 кВ в режимі максимальних та мінімальних навантажень. Зазначено, що на добовому інтервалі процес нестационарний. На добовому графіку електричного навантаження виділено годинні інтервали часу, проведено дослідження закону розподілу, за критерієм Пірсона підтверджено нормальний закон розподілу. Виконана перевірка стаціонарності за параметричними тестами, та кореляційною функцією.

На ділянках стаціонарності отримані значення числових характеристик та побудована ймовірнісна математична модель навантаження ТП 10/0,4 кВ.

Розроблено метод вирівнювання графіка електричного навантаження ТП 10/0,4 кВ шляхом оптимального керування потужністю споживачів-регуляторів у міських електричних мережах з врахуванням особливостей графіку електричного навантаження основних споживачів ТП 10/0,4 кВ. З цією метою задана цільова функція, де критерієм оптимізації прийнято сумарний мінімум вартості на покриття втрат електричної енергії в мережі викликаних нерівномірністю графіку електричного навантаження та за спожиту електричну енергію, складена система обмежень обумовлених навантажувальною здатністю трансформаторів і необхідною кількістю електричної енергії на виконання технологічного процесу споживачами-регуляторами, та граничні умови обумовлені встановленими потужностями споживачів-регуляторів. Для реалізації процесу оптимізації використаний метод нелінійного програмування, знаходження екстремуму функції здійснено за методом Ньютона. Рішення задачі реалізовано у MS Excel.

Ключові слова - графік електричного навантаження, розподільні електричні мережі, ймовірнісна математична модель, нелінійне програмування, оптимізація, споживачі-регулятори